FLOW OF A SLIGHTLY RAREFIED GAS IN A NARROW SLIT CHANNEL IN THE CASE OF SUBLIMATION FROM ONE OF THE WALLS AND HEAT INFLOW FROM THE OTHER

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The flow of a slightly rarefied gas (Kn < 0.1) in a narrow slit channel during sublimation from one of the walls under the influence of an inflow of heat (by conduction and radiation) from the other wall of the channel is considered. An equation is derived for the pressure distribution in narrow slit channels in a form convenient for analytical investigation.

Various cases of the flow of a slightly rarefied vapor through narrow slit channels were studied in [1-3] for low and medium Knudsen numbers (0.01-0.1) with symmetrical sublimation from both walls. One characteristic of these flows was the practically uniform distribution of the thermodynamic parameters of the vapor along the normal to the wall.

In many technological devices (such as sublimation chambers of the tray variety) the heat- and mass-transfer processes are not organized symmetrically; this distorts the uniformity of the vapor-flow parameter distribution and, correspondingly, modifies the character of the flow. In order to analyze this influence we shall here consider flows of slightly rarefied vapor in a narrow gap between parallel walls (Fig. 1) arising as a result of sublimation from the wall 1 into the channel. We shall assume that heat is conveyed to the phase-transition surface from wall 2 by way of the flowing vapor and radiation.

In the symmetry plane of the channel we introduce the rectangular coordinates 0xy. The distance from this plane is measured by the coordinate z. If the height of the slit channel 2h($-h < z < h$) is small compared with the scale L of vapor flow in the 0xy plane, the equations of motion, continuity, and energy may conveniently be expressed in the following form (as in the analysis of the flows considered in [1-4] we shall specially distinguish the transverse velocity component ($\mathbf{U} = \mathbf{U} + kw$) and the derivatives with respect to z):

$$\frac{\partial}{\partial z}\left(\mu \frac{\partial \mathbf{u}}{\partial z}\right) - \nabla_1 P = \rho (\mathbf{u} \nabla_1) \mathbf{u} + \rho w \frac{\partial \mathbf{u}}{\partial z} - \nabla_1 (\mu \nabla_1 \mathbf{u}); \tag{1}$$

$$\frac{\partial P}{\partial z} = \frac{\partial}{\partial z}\left(\mu \frac{\partial w}{\partial z}\right) - \rho w \frac{\partial w}{\partial z} - \rho \mathbf{u} \nabla_1 w + \nabla_1 (\mu \nabla_1 w); \tag{2}$$

$$\frac{\partial}{\partial z}\left(\mu \frac{\partial H}{\partial z}\right) - \Pr\left[P \nabla_1 \mathbf{u} - \mu \left(\frac{\partial u}{\partial z}\right)^2\right] = \Pr\left[\rho \mathbf{u} \nabla_1 H + \rho w \frac{\partial H}{\partial z} + O\left(\mu \frac{\partial u}{\partial z} |\nabla_1 w|\right) + O(\mu |\nabla_1 \mathbf{u}|^2) + O\left(\mu \left|\frac{\partial w}{\partial z}\right|^2\right)\right] + \nabla_1 (\mu \nabla_1 H); \tag{3}$$

$$\frac{\partial (\rho w)}{\partial z} + \Delta_1 (\rho \mathbf{u}) = 0. \tag{4}$$

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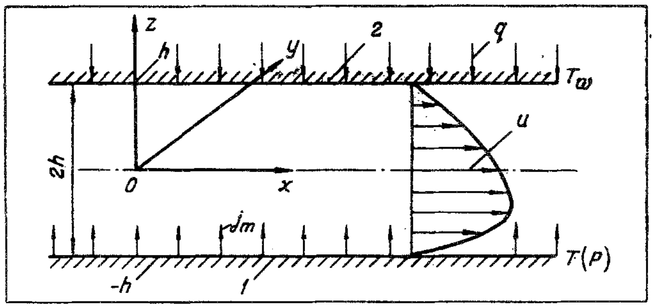

Fig. 1. Scheme of flow of a slightly rarefied vapor in the gap between parallel walls.

In view of the possible severe inhomogeneity of the temperature field across the slit channel, we allow for the temperature dependence of the coefficient μ, but assume that c_p and Pr are constants.

At the walls of the channel we have to satisfy conditions corresponding to slippage and a temperature jump. If we neglect the thermal creep and the resistance of the phase transition (shown to have only a slight effect on the flows under consideration in [2]) and also quantities of the order of h^2/L^2 by comparison with unity, these conditions may be written in the form [5]

$$\left.\begin{aligned} u &= \frac{2-\theta}{\theta}\sqrt{\gamma\frac{\pi}{2}}\,\frac{\mu}{\rho\sqrt{(\gamma-1)H}}\,\frac{\partial u}{\partial z}, \\ \frac{H}{H(P)} &= 1 + \frac{2-\alpha}{\alpha}\,\frac{15}{8}\sqrt{\gamma\frac{\pi}{2}}\,\frac{\mu}{H\rho\sqrt{(\gamma-1)H}}\,\frac{\partial H}{\partial z}\end{aligned}\right\}\ \text{for } z=-h; \tag{5}$$

$$\left.\begin{aligned} u &= -\frac{2-\theta}{\theta}\sqrt{\gamma\frac{\pi}{2}}\,\frac{\mu}{\rho\sqrt{(\gamma-1)H}}\,\frac{\partial u}{\partial z}, \\ \frac{H}{H_w} &= 1 - \frac{2-\alpha}{\alpha}\,\frac{15}{8}\sqrt{\gamma\frac{\pi}{2}}\,\frac{\mu}{H\rho\sqrt{(\gamma-1)H}}\,\frac{\partial H}{\partial z}\end{aligned}\right\}\ \text{for } z=h. \tag{6}$$

We shall consider laminar vapor flows corresponding to small Reynolds numbers ($Re = \rho V_1 h^2/\mu L$). Since the terms on the right-hand sides of Eqs. (1)-(3), considered in relation to the terms on the left-hand sides of the corresponding equations, are quantities of the order of Re or h^2/L^2, in the linear approximation (to which we shall here restrict our analysis) these may be neglected; taking the power relationship of [4] for the viscosity [$\mu_w/\mu_{w0} = (H/H_{w0})^{\omega}$] ($\omega \approx 0.8$ for $T \approx 273°K$; $\omega \approx 1$ for $T \ll 273°K$), we may then write system (1)-(3) and conditions (5), (6) in dimensionless form as follows:

$$\frac{\partial}{\partial\zeta}\left(\chi^{\omega}\frac{\partial U}{\partial\zeta}\right) = \nabla\Pi;\qquad \frac{\partial\Pi}{\partial\zeta} = 0; \tag{7}$$

$$U = -k_u\frac{\chi^{\omega+1/2}}{\Pi}\,\frac{\partial U}{\partial\zeta};\qquad \chi = \chi_w - k_H\chi_w\frac{\chi^{\omega-1/2}}{\Pi}\,\frac{\partial\chi}{\partial\zeta}. \tag{8}$$

for $\zeta = -1$

$$\frac{\partial}{\partial\zeta}\left(\chi^{\omega}\frac{\partial\chi}{\partial\zeta}\right) = (\gamma-1)\Pr M^2\left[\Pi\nabla U - \chi^{\omega}\left(\frac{\partial U}{\partial\zeta}\right)^2\right]; \tag{9}$$

for $\zeta = 1$

$$U = k_u\frac{\chi^{\omega+1/2}}{\Pi}\,\frac{\partial U}{\partial\zeta};\quad \chi = \chi(\Pi) + k_H\chi(\Pi)\frac{\chi^{\omega-1/2}}{\Pi}\,\frac{\partial\chi}{\partial\zeta}; \tag{10}$$

The problem represented by Eqs. (7)-(10) is similar to that considered in [5], which related to Couette flow in a slightly rarefied gas; it differs from the latter chiefly by virtue of the nonzero pressure gradient $\nabla\Pi$ and the consequent field of velocities averaged over the height of the slit channel, which is characterized by the linear scale L ($L \gg h$). In addition to this, in the present case there is no relative motion of the walls, and a different approximation is used for the temperature dependence of the coefficient μ.

If we confine attention to fairly slow subsonic flows ($M^2 \ll 1$), the right-hand side of Eq. (8) may be neglected. Integrating Eqs. (7) and (8) with respect to ζ a corresponding number of times we deduce

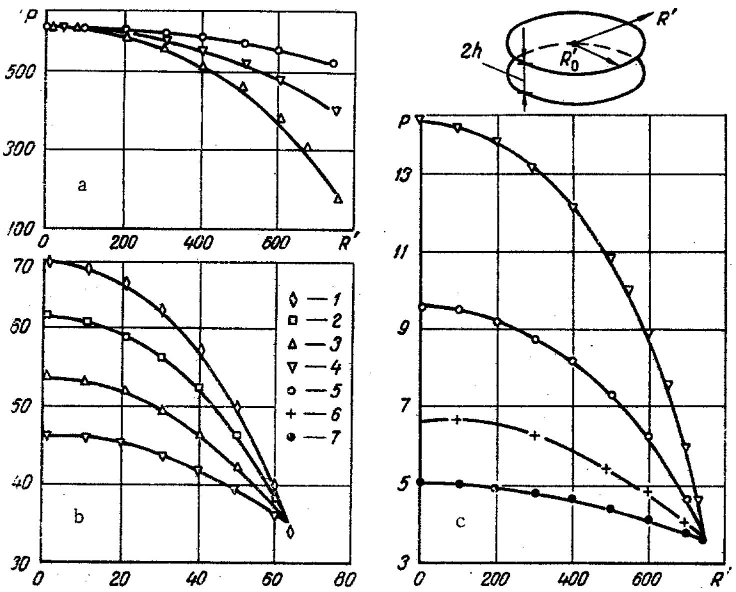

Fig. 2. Change of pressure (N/m^2) in the gap between circular disks (R', mm) for a sublimation rate j_m (kg/m^2 · sec) equal to: 1) $10 \cdot 10^{-4}$; 2) $8 \cdot 10^{-4}$; 3) $6 \cdot 10^{-4}$; 4) $4 \cdot 10^{-4}$; 5) $2 \cdot 10^{-4}$; 6) $1 \cdot 10^{-4}$; 7) $0.5 \cdot 10^{-4}$ and a slit height 2h (mm) of: a) 2; b) 2; c) 20.

$$\mathbf{U} = \frac{\mathbf{B}_1}{b_2}[\chi(\zeta) - \chi(-1)] + \left[\frac{\zeta\chi(\zeta) + \chi(-1)}{b_2} - \frac{\chi^{\omega+2}(\zeta) - \chi^{\omega+2}(-1)}{(\omega+2)b_2^2}\right]\nabla\Pi + \mathbf{B}_4; \quad \Pi = \Pi(\xi, \eta); \tag{11}$$

$$\chi(\zeta) = [b_3 + (\omega+1)b_2\zeta]^{1/(\omega+1)}. \tag{12}$$

Here $\mathbf{B}_k = \mathbf{B}_k(\xi\ \eta)$; $b_j = b_j(\xi, \eta)$; $\chi(\zeta) = \chi(\xi, \eta, \zeta)$.

Since we are considering flows corresponding to Knudsen numbers of Kn ≤ 0.1, on substituting (2) into the boundary conditions (9) and (10) we may neglect quantities of the order of k_H^2 [$k_H = O(\mathrm{Kn})$] and make the substitution $\chi(-1) \approx \chi(\Pi)$, $\chi(1) \approx \chi_w$ on the right-hand sides. We thus obtain

$$b_2 = \frac{\chi_w^{\omega+1} - \chi^{\omega+1}(\Pi)}{(\omega+1)\left\{2 + \frac{k_H}{\Pi}[\chi_w^{\omega+1/2} + \chi^{\omega+1/2}(\Pi)]\right\}};$$

$$b_3 = \chi_w^{\omega+1} - \frac{(\Pi + k_H\chi_w^{\omega+1/2})[\chi_w^{\omega+1} - \chi^{\omega+1}(\Pi)]}{2\Pi + k_H[\chi_w^{\omega+1/2} + \chi^{\omega+1/2}(\Pi)]}; \tag{13}$$

$$\chi(-1) = \chi(\Pi) + \frac{k_H[\chi_w^{\omega+1} - \chi^{\omega+1}(\Pi)]\chi^{1/2}(\Pi)}{2(\omega+1)\Pi};$$

$$\chi(1) = \chi_w - \frac{k_H[\chi_w^{\omega+1} - \chi^{\omega+1}(\Pi)]\chi_w^{1/2}}{2(\omega+1)\Pi}.$$

Working to the same accuracy, by substituting (12) into the boundary conditions (9) and (10) we obtain $\mathbf{B}_k = b_k\nabla\Pi$ where

$$b_1 = \frac{[\chi^{\omega+2}(+1) - \chi^{\omega+2}(-1)]\Pi - (\omega+2)b_2\{[\chi(1) + \chi(-1)]\Pi + k_u b_2[\chi_w^{1/2} - \chi^{1/2}(\Pi)]\}}{(\omega+2)b_2\{[\chi(1) - \chi(-1)]\Pi + k_u b_2[\chi_w^{1/2} + \chi^{1/2}(\Pi)]\}};$$

$$b_4 = k_u(b_1 - 1)\chi^{1/2}(\Pi)/\Pi. \tag{14}$$

After integrating Eq. (4) with respect to z between −h and h, allowing for the fact that sublimation takes place from the lower wall [$w(h) = 0$; $\rho(-h)w(-h) = j_m$], we obtain a differential equation describing the pressure distribution in the part of the slit channel under consideration:

$$\nabla\mathbf{U}_1 = \frac{\gamma - 1}{\gamma}\frac{hH_{w0}}{\mu_{w0}V_1^2}j_m; \quad \mathbf{U}_1 = \Pi\int_{-1}^{1}\frac{Ud\zeta}{\chi}. \tag{15}$$

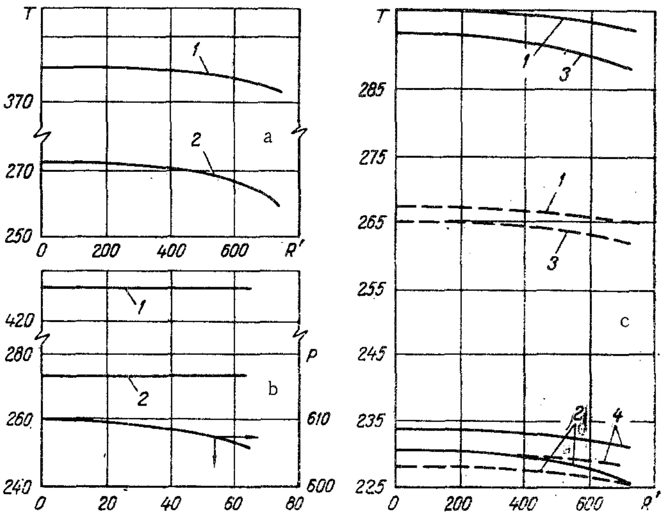

Fig. 3. Temperature distribution in the gap between circular disks (R', mm): 1) temperature of heater disk T_W; 2) subliming disk T(P); 3, 4) vapor temperature T(h) and T(—h) in the gap between the disks for a slit height 2h (mm) and a sublimation rate j_m (kg/m^2 · sec) of: a) 2 and $6\cdot 10^{-4}$; b) 2 and $10\cdot 10^{-4}$; c) 20 and $1\cdot 10^{-4}$ (continuous lines) or $0.5\cdot 10^{-4}$ (broken lines).

In general,

$$U_1 = \frac{1}{2} A(\Pi, \chi_w) \nabla \Pi^2,$$

where

$$A(\Pi, \chi_w) = 2\frac{b_1}{b_2} + \left[\frac{(1-b_1)\chi(-1)}{\omega b_2^2} + \frac{\chi^{\omega+2}(-1)}{\omega(\omega+2)b_2^3} + \right.$$

$$\left. + \frac{b_4}{\omega b_2}\right][\chi^{\omega}(1) - \chi^{\omega}(-1)] - \frac{\chi^{2(\omega+1)}(1) - \chi^{2(\omega+1)}(-1)}{2(\omega+1)(\omega+2)b_2^3}.$$

The sublimation intensity j_m is related to the specific thermal flux q traveling toward the lower wall by the obvious equation $j_m = q/r$. The flux $q = q_1 + q_2$ where q_1 is the heat flow from the vapor, and q_2 is the radiant thermal flux absorbed by the phase-transition surface.

It is already well known [5, 6] that for the flow of a rarefied gas q_1 consists not only of the flow of heat associated with thermal conductivity, but also that due to the work of frictional forces, i.e.,

$$q_1 = \left(\lambda \frac{\partial T}{\partial z} + \mu u \frac{\partial u}{\partial z}\right)_{z=-h} = \frac{\lambda_{w0} T_{w0}}{h}\left[\chi^{\omega}\frac{\partial \chi}{\partial \zeta} + (\gamma-1)\,\mathrm{Pr}\,M^2 \chi^{\omega} U \frac{\partial U}{\partial \zeta}\right].$$

Since we are considering fairly slow flows ($M^2 \ll 1$), to the accuracy already specified we may neglect the second component of heat flux as being a quantity of the order of $\mathrm{Kn}M^2$. By virtue of (12)

$$q_1 = \frac{\lambda_{w0} T_{w0}}{h} b_2.$$

If we neglect the absorption of the radiant flux in the narrow slit channel and remember that the linear scale (characterizing the essential changes in the flow parameters and the temperature T_W of the heated wall) $L \gg h$, we may consider that radiant heat transfer takes place almost solely between opposite elements of the channel walls:

$$q_2 = \varepsilon\sigma[T_w^4 - T^4(P)] = \varepsilon\sigma T_{w0}^4[\chi_w^4 - \chi^4(\Pi)].$$

By substituting the expressions for j_m, q_1, q_2 into (15) and using the equations

$$(\gamma-1)H_{w0}/V_1^2 = M^{-2}; \quad c_p\mu_{w0} = \mathrm{Pr}\,\lambda_{w0},$$

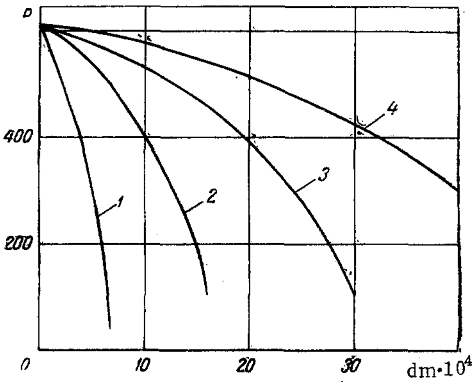

Fig. 4. Dependence of the limiting permissible pressures (P, N/m^2) at the cutoff point of the slit channel between the disks on the intensity of sublimation j_m (kg/m^2·sec) and the slit height 2h (mm): 1) 2; 2) 3; 3) 4; 4) 5.

we thus obtain the fundamental equation

$$\nabla [A(\Pi, \chi_w)\nabla \Pi^2] = \frac{2}{\gamma M^2 \Pr \Lambda(\Pi)} \Big\{ \mathrm{Bi}\,[\chi_w^4 - \chi^4(\Pi)] + \frac{\chi_w^{\omega+1} - \chi^{\omega+1}(\Pi)}{(\omega+1)\{2 + k_{\text{н}}[\chi_w^{\omega+1/2} + \chi^{\omega+1/2}(\Pi)]\Pi\}}. \tag{16}$$

In any practical systems, either the thermal load distribution q(x, y) along the heated wall or the corresponding temperature distribution $T_W(x, y)$, $[\chi_W = \chi_W(\xi, \eta)]$ may be specified. If the transfer processes are limited by the access of the sublimed material to the surface z = −h, the specified quantity is the function $j_m(x, y)$. In those cases in which we may neglect the dependence of the latent heat of the phase transition on the parameters of the vapor flowing around the element of the subliming surface [r(P) ≈ const], the specification of $j_m(x, y)$ is equivalent to the specification of q(x, y).

For specified distributions of $j_m(x, y)$ [or q(x, y)], the dimensionless enthalpy of the heated wall χ_W may be found from the following transcendental equation:

$$\frac{\chi_w^{\omega+1} - \chi^{\omega+1}(\Pi)}{2(\omega+1)} \left\{1 - \frac{k_{\text{н}}}{2\Pi}[\chi_w^{\omega+1/2} + \chi^{\omega+1/2}(\Pi)]\right\} + \mathrm{Bi}\,[\chi_w^4 - \chi^4(\Pi)] = \Omega. \tag{17}$$

Here $\Omega = \Pr h r(P) j_m(x, y)/\mu_{W0}$ or $\Omega = \Pr h q(x, y)/\mu_{W0}$. The solution is a function of the form $\chi_W = \chi_W(\Pi, \xi, \eta)$. In a number of cases this function is easy to express in analytical form. For Bi ≫ 1 (radiant heat transfer predominant) to a first approximation

$$\chi_w = \sqrt[4]{\chi^4(\Pi) + \frac{\Omega}{\mathrm{Bi}}}. \tag{18}$$

This solution may be refined by the perturbation method, making use of the discarded terms of Eq. (17). The parameter Ω under the root sign is then replaced by the parameter Ω_1:

$$\Omega_1 = \Omega - \frac{\left[\chi^4(\Pi) + \frac{\Omega}{\mathrm{Bi}}\right]^{(\omega+1)/4} - \chi^{\omega+1}(\Pi)}{2(\omega+1)} \times \left\{1 - \frac{k_{\text{н}}}{2\Pi}\left[\left[\chi^4(\Pi) + \frac{\Omega}{\mathrm{Bi}}\right]^{(\omega+1/2)/4} + \chi^{\omega+1/2}(\Pi)\right]\right\}.$$

It is also easy to obtain a solution to Eq. (17) for Bi ≪ 1. If T_W ≪ 273°K we have ω ≈ 1. In this case Eq. (17) differs from the biquadratic form by a perturbing term of the order of $k_H = O(\mathrm{Kn})$. The solution may easily be obtained to an accuracy of O(Kn) by perturbation theory.

For a specified distribution of $j_m(x, y)$, and also for r = const and a specified q(x, y) distribution, we may express Eq. (16) in the form

$$\nabla [A_1(\Pi, \xi, \eta)\nabla \Pi^2] = \Phi(\xi, \eta). \tag{19}$$

Here

$$A_1(\Pi, \xi, \eta) = A[\Pi, \chi_w(\Pi, \xi, \eta)]; \quad \Phi(\xi, \eta) = \frac{h j_m}{\mu_{w0}\gamma M^2}.$$

For flows along the x axis and flows possessing cylindrical symmetry (slit channel formed by two disks $\Phi(\xi, \eta) = \Phi_1(R')$, $R' = \sqrt{\xi^2 + \eta^2}$) the equation of motion of the rarefied gas (19) reduces to an ordinary differential equation,

$$\frac{1}{(R')^\nu}\frac{d}{dR'}\left[(R')^\nu A_2(\Pi, R')\frac{d\Pi^2}{dR'}\right] = \Phi_1(R'). \tag{20}$$

Here $A_2(\Pi, R') = A_1(\Pi, \xi, \eta)$. The parameter ν is equal to 0 or 1 for flows along the x axis and those with cylindrical symmetry, respectively. In the first case R' = ξ.

In practice it is usual to specify the pressure at the cutoff point of the slit channel $\Pi(R_0')$ (for ν = 0 R_0' is the length of the slit channel; for ν = 1 R_0' is the radius of the disks forming the walls of the slit channel). For R' = 0 dΠ/dR' = 0.

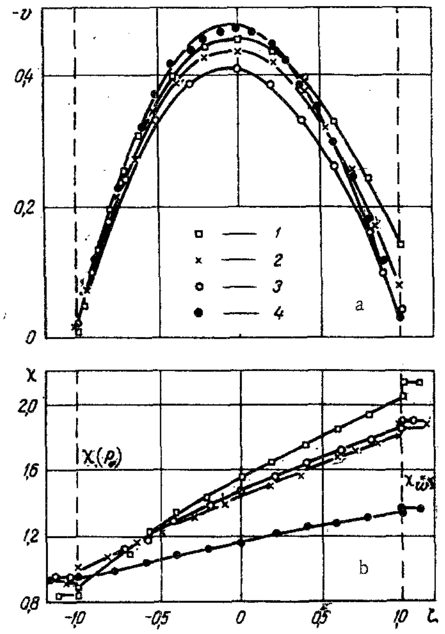

Fig. 5. Velocity (a) and temperature $\chi = T/T_{w0}$ ($T_{w0} = 273°K$) (b) profiles of the flow of a slightly rarefied gas in narrow slit channels for the following values of 2h (mm), P (n/m²), and j_m (kg/m² · sec), respectively: 1) 20; 7.2; $2 \cdot 10^{-3}$; 2) 2; 100; $2 \cdot 10^{-3}$; 3) 2; 176.96; $2 \cdot 10^{-3}$; 4) 2; 176.96; $6 \cdot 10^{-4}$. $-v = \mu_{w0} U / h^2 \nabla P$; $\zeta = z/h$; $\chi = T/T_{w0}$.

The first integral of Eq. (20) satisfying the latter condition is

$$\frac{d\Pi}{dR'} = \frac{\Phi_2(R')}{(R')^{\nu} A_2(\Pi, R')}; \quad \Phi_2(R') = \int_0^{R'} (R_1')^{\nu} \Phi_1(R_1')\, dR_1'. \qquad (21$$

Thus, the pressure distribution in the slit channel is found by solving the Cauchy problem for Eq. (21) by reference to the specified pressure Π for $R' = R_0'$, which may easily be done by the Runge — Kutta method in an electronic computer.

An equation extremely convenient in any analytical investigation of the process is obtained for the case of uniform sublimation from the lower wall (j_m = const), which may be realized either by introducing a corresponding supply of the subliming material or by the uniform inflow of heat to the heated wall (q = const) if $r \approx$ const. In this case the solution of Eq. (17) takes the form $\chi_w = \chi_w(\Pi)$ and hence $A(\Pi, \chi_w) = A_3(\Pi)$. It is, furthermore, quite clear that $\Phi(\xi, \eta) = \Phi_3$ = const. Thus, the basic equation (16) here degenerates into the Poisson equation

$$\nabla^2 \Psi = \Phi_3; \quad \Psi = 2 \int^{\Pi} \Pi A_3(\Pi)\, d\Pi, \qquad (22)$$

and we may then make use of various powerful methods of the theory of analytical functions in order to find the pressure distribution in slit channels bounded by a contour Γ having a fairly complicated configuration, containing arcs Γ_j corresponding to construction elements which impede the free egress of vapor ($\partial \Pi / \partial n = 0$ on Γ_j). For example, the results of the problems solved in [2, 3] may easily be extended to the processes here under consideration. Here it is convenient to return to dimensional quantities. Instead of (22) we then obtain

$$\nabla_1 \Psi_1 = \frac{2\mu j_m}{3h^3}; \quad \Psi_1 = -\frac{2}{3} \frac{1}{RT_{w0}} \int^{P} P A_3(P)\, dP. \qquad (22')$$

Here

$$A_3(P) = 2\frac{b_1}{b_2} + \left[\frac{(1-b_1)\chi_-}{\omega b_2^2} + \frac{\chi_-^{\omega+2}}{\omega(\omega+2) b_2^3} - \frac{b_4}{\omega b_2}\right](\chi_+^{\omega} - \chi_-^{\omega}) - \frac{\chi_+^{2(\omega+1)} - \chi_-^{2(\omega+1)}}{2(\omega+1)(\omega+2) b_2^3};$$

$$b_1 = \frac{\chi_+^{\omega+2} - \chi_-^{\omega+2} - (\omega+2) b_2 \{\chi_+ - \chi_- + k_u b_2 [\chi_w^{1/2} - \chi^{1/2}(P)]\}}{(\omega+2) b_2 \{\chi_+ - \chi_- + k_u b_2 [\chi_w^{1/2} + \chi^{1/2}(P)]\}};$$

$$b_2 = \frac{\chi_w^{\omega+1} - \chi^{\omega+1}(P)}{(\omega+1)\{2 + k_H [\chi_w^{\omega+1/2} + \chi^{\omega+1/2}(P)]\}};$$

$$b_4 = (b_1 - 1) k_u \sqrt{\chi(P)};$$

$$\chi_+ = \chi_w - \frac{k_H}{2} \frac{\chi_w^{\omega+1} - \chi^{\omega+1}(P)}{\omega+1} \sqrt{\chi_w};$$

$$\chi_- = \chi(P) + \frac{k_H}{2} \frac{\chi_w^{\omega+1} - \chi^{\omega+1}(P)}{\omega+1} \sqrt{\chi(P)};$$

$$k_u = \frac{2-\theta}{\theta} \sqrt{\gamma \frac{\pi}{2}} \frac{\mu_{w0} R \sqrt{T_{w0}}}{\sqrt{(\gamma-1) c_p}\, hP};$$

$$k_H = \frac{2-\alpha}{\alpha} \frac{15}{8} \sqrt{\gamma \frac{\pi}{2}} \frac{\mu_{w0} R \sqrt{T_{w0}}}{\sqrt{(\gamma-1) c_p}\, hP}.$$

For purposes of illustration Figs. 2 and 3 show the pressure and temperature distributions in the gaps between circular disks, calculated by means of the equation

$$\Psi = \Psi(R_0') + \frac{\mu j_m}{6h^3} [(R_0')^2 - (R')^2]$$

for the sublimation of ice. It was assumed in these calculations that $\theta = \alpha = 1$; $\varepsilon_0 = 0.9$; $\omega = 0.8$; $\gamma = 1.3505$; Pr = 0.846.

Of particular interest are those cases of sublimation in which the pressure in the center of the disk approaches the value corresponding to the triple point ($P = 610.67\ N/m^2$ for ice, Fig. 2a).

The curves shown in Fig. 4 provide an upper limit to the zone of permissible conditions, beyond which melting of the ice begins in the central part of the disk. The vertical axis gives the pressure at the cutoff point of the slit channel ($R' = R'_0 = 0.75$ m).

Considering these velocity and temperature profiles (Fig. 5), we see that with falling pressure of the medium and the same rate of sublimation the slip velocity and temperature jump increase (curves 1, 2, 3). Thus, for example, when $P = 7.2\ N/m^2$ the temperature jump is 30° and when $P = 176.96\ N/m^2$ — 2.7°. For a pressure of over $610.67\ N/m^2$ the slip velocities and temperature jump are so small that they may be neglected.

If the temperature distribution is specified on the heated wall

$$T_w = T_w(x, y), \quad [\chi_w = \chi_w(\xi, \eta)],$$

we have $A(\Pi, \chi_w) = A_4(\Pi, \xi, \eta)$ and the fundamental Eq. (16) reduces to the form

$$\nabla [A_4(\Pi, \xi, \eta) \nabla \Pi^2] = \Phi_4(\Pi, \xi, \eta). \tag{23}$$

For flows along the x axis ($\xi = R'$, $\nu = 0$) and flows with cylindrical symmetry ($R' = \sqrt{\xi^2 + \eta^2}$, $\nu = 1$) the latter equation degenerates into an ordinary differential equation,

$$\frac{1}{(R')^{\nu}} \frac{d}{dR'} \left[(R')^{\nu} A_5(\Pi, R') \frac{d\Pi^2}{dR'} \right] = \Phi_5(\Pi, R'),$$

$$\frac{d\Pi}{dR'} = 0 \text{ for } R' = 0, \quad \Pi = \Pi_0 \text{ for } R' = R'_0.$$

When studying the sublimation process over a fairly wide range of pressures, it is desirable to replace the solution of the nonlinear two-point problem (24)-(25) by the solution of the Cauchy problem which we obtain if we specify the vapor pressure Π for $R' = 0$. The solution of the latter by the Runge—Kutta method is easily carried out in an electronic computer.

NOTATION

P, H, ρ and μ	are the pressure, enthalpy, density, and dynamic viscosity of the vapor;
u w	are the projections of the flow velocity on the symmetry plane of the channel and the normal to the wall;
$H(P)$ and H_W	are the enthalpy of the saturated vapor at a pressure P and at the temperature of the heated wall, respectively;
V_1, L	are the characteristic velocity and scale in the symmetry plane;
q, q_1, q_2	are the total specific heat flux in the direction of the sublimation surface and components corresponding to inflow from the vapor and radiant flux;
σ, ε	are the Stefan—Boltzmann constant and the reduced emissivity of opposite elements of the channel walls;
α, θ	are the accommodation coefficient and proportion of diffusely reflected molecules;
c_p, c_v	are the specific heats of the vapor at constant pressure and constant volume, respectively;
r,	is the latent heat of sublimation;
$0xy$ and z	are the rectangular coordinates in the symmetry plane and distance from this plane;
j_m	is the rate of sublimation;

$k_H = (2-\alpha)/\alpha \cdot (15/8)\sqrt{\gamma\pi/2}(M/Re_h)$; $k_u = (2-\theta)/(\theta)\sqrt{\gamma\pi/2}(M/Re_h)$; $Re_h = (P_0V_1h)/(\mu_{W0}RT_{W0})$; $P_0 = \mu_{W0}V_1L/h^2$; $Re = \rho V_1 h^2/(\mu L)$;

Kn, Pr	are the Knudsen and Prandtl numbers;
$M = V_1/[(\gamma-1)H_{W0}]^{1/2}$	is the characteristic Mach number;
$\gamma = c_p/c_v$; $T_{W0} = H_{W0}/c_p$;	$Bi = h\varepsilon\sigma T_{W0}^3/\lambda_{W0}$; λ_{W0}; μ_{W0} are the thermal conductivity and dynamic viscosity of the vapor at temperature T_{W0}. The index 0 refers to parameters at the cutoff point of the slit channel.

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